

HW4, due to 5/10/2021

Exercise 1. (1) Let Γ be the subgroup of $\mathbf{SL}_2(\mathbf{Z})$ generated by

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Prove $\Gamma = \mathbf{SL}_2(\mathbf{Z})$.

(2) For any $\gamma, \gamma' \in \mathbf{SL}_2(\mathbf{Z})$ and any $\tau \in \mathbf{H}$, one has

$$(\gamma \cdot \gamma')(z) = \gamma(\gamma'(z)).$$

(3) Prove

$$\mathbf{Im}(\gamma(\tau)) = \frac{\mathbf{Im}(\tau)}{|c\tau + d|^2}, \quad \frac{d}{d\tau} \gamma(\tau) = \frac{1}{(c\tau + d)^2}, \quad \gamma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{SL}_2(\mathbf{Z}).$$

Exercise 2. Exercise 1.20, Page 46, in the Lecture Note.

Exercise 3. Exercise 1.22 (verify (2) and (5)), Page 53, in the Lecture Note.

Exercise 4. Verify (1.3.3.15) in the Lecture Note.