HW4, due to 11/2/2020

Exercise 1. (1) Let Γ be the subgroup of $\mathbf{SL}_2(\mathbf{Z})$ generated by

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Prove $\Gamma = \mathbf{SL}_2(\mathbf{Z})$.

(2) For any $\gamma, \gamma' \in \mathbf{SL}_2(\mathbf{Z})$ and any $\tau \in \mathbf{H}$, one has

$$(\gamma \cdot \gamma')(z) = \gamma(\gamma'(z)).$$

(3) Prove

$$\mathbf{Im}(\gamma(\tau)) = \frac{\mathbf{Im}(\tau)}{|c\tau + d|^2}, \quad \frac{d}{d\tau}\gamma(\tau) = \frac{1}{(c\tau + d)^2}, \quad \gamma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{SL}_2(\mathbf{Z}).$$

Exercise 2. Verify (1.3.3.14) in the Lecture Note.

Exercise 3 (Bonus). Check δ^p in Page 50 of the Lecture Note is independent of the choice of a, b, c.