HW4, due to 11/7

1. Page 111, Exercise 4, 5, 8.

2. Page 112, Exercise 13.

3. Page 118, Exercise 2, 3, 8.

4. Page 127, Exercise 4, 6.

5. Page 133, Exercise 3.

* **Bonus**
*1. Page 129, Exercise 11.

*2. Let (X, d) be a metric space. For any two subsets A, B of X, define its **Hausdorff distance** by

$$d_{\mathbf{H}}(A, B) := \inf\{\epsilon > 0 : A \subset B_\epsilon \text{ and } B \subset A_\epsilon\}.$$

Here $A_\epsilon := \{x \in A : d(x, A) < \epsilon\}$ and $d(x, A) := \inf\{d(x, a) : a \in A\}$. Show that $d_{\mathbf{H}}$ is a **semi-metric** on 2^X (the set of all subsets of X), i.e.,

- $d_{\mathbf{H}}(A, B) \geq 0$, for any $A, B \in 2^X$.
- $d_{\mathbf{H}}(A, B) = d_{\mathbf{H}}(B, A)$, for any $A, B \in 2^X$.
- $d_{\mathbf{H}}(A, C) \leq d_{\mathbf{H}}(A, B) + d_{\mathbf{H}}(B, C)$, for any $A, B, C \in 2^X$.

Furthermore,

(1) Show that $d_{\mathbf{H}}(A, \overline{A}) = 0$ for any $A \in 2^X$, where $\overline{A}$ denotes the closure of A.

(2) Show that $d_{\mathbf{H}}(A, B) = 0$ implies $A = B$, where A, B are closed subsets in the metric topology induced by d. Hence $(\mathcal{X}, d_{\mathbf{H}})$ is a metric space, where $\mathscr{X}$ denotes the set of closed subsets of X relative to the metric topology induced by d.