

HW6, due to 6/2/2021

Exercise 1. Show that $\mathbf{O}(n)$ is compact but $\mathbf{GL}(n, \mathbf{R})$ is noncompact.

Exercise 2. If $f : (X, \mathcal{T}_X) \rightarrow (Y, \mathcal{T}_Y)$ is continuous and X is path-connected, then $f(X)$ is also path-connected.

Exercise 3. Any finite product of path-connected spaces is path-connected.

Exercise 4. Show that \mathbf{Q} is neither connected nor locally connected.

Exercise 5 (bonus). In the class, we showed that

$$\ln(e^X e^Y) = X + Y + \frac{1}{2}[X, Y] + \dots$$

for any $X, Y \in \mathbf{Max}(n, \mathbf{C})$ with $\|X\|$ and $\|Y\|$ sufficiently small. Please verify

$$\ln(e^X e^Y) = X + Y + \frac{1}{2}[X, Y] + \frac{1}{12}[X, [X, Y]] - \frac{1}{12}[Y, [X, Y]] + \dots$$